## Instructions:

- This exam is closed book.
- Make sure you sign your exam booklet. Failure to do so will result in a 2 points deduction on your exam grade.
- The exam duration is $\mathbf{9 0}$ minutes.
- Answer all questions on your exam booklet, starting each problem on a new page. You may annotate the figures in the question sheet, but make sure you refer to them in your answer.
- State your assumptions and show your work leading to the final answer.
- Return this exam question sheet with your exam booklet.


## Good luck!

## Problem 1: (25 points)

$$
\frac{u}{U}=\frac{3}{2} \frac{y}{\delta}-\frac{1}{2}\left(\frac{y}{\delta}\right)^{3}
$$



Fixed surface

A Newtonian fluid having a specific gravity of 0.92 and a kinematic viscosity of $4 \times 10^{-4} \mathrm{~m}^{2} / \mathrm{s}$ flows past a fixed surface. Due to the no-slip condition, the velocity at the fixed surface is zero, and the velocity profile near the surface shown in the figure is described by the following equation:

$$
u=U\left\{\frac{3}{2} \frac{y}{\delta}-\frac{1}{2}\left(\frac{y}{\delta}\right)^{3}\right\}
$$

where $u$ is the velocity ( $\mathrm{m} / \mathrm{s}$ ), $U$ the velocity ( $\mathrm{m} / \mathrm{s}$ ) at thickness $\delta(\mathrm{m})$, and y the position on the vertical axis.
Determine the magnitude of the shearing stress developed on the plate surface ( $\mathrm{y}=0$ ). Express your answer in terms of U and $\delta$.

## Problem 2: (25 points)



As water rises on the left side of the rectangular gate with two sides in contact with water, the gate will open automatically.
At what depth above the hinge will this occur? Neglect the mass of the gate.

## Problem 3: (25 points)



A container with two circular vertical tubes of diameters $d_{1}=39.5 \mathrm{~mm}$ and $\mathrm{d}_{2}=$ 12.7 mm is partially filled with mercury (density $13550 \mathrm{~kg} / \mathrm{m}^{3}$ ). The equilibrium level of the liquid is shown in the left diagram. A cylindrical object made from solid brass (density $8550 \mathrm{~kg} / \mathrm{m}^{3}$ ) is placed in the larger tube so that is floats, as shown in the right diagram. The object is $\mathrm{D}=37.5 \mathrm{~mm}$ in diameter and $\mathrm{H}=76.2$ mm high. Neglect any capillary rise due to surface tension effects.
(a) Calculate the pressure at the lower surface needed to float the object (20 points)
(b) Determine the new equilibrium level, $h$, of the mercury with the brass cylinder in place (5 points)

## Problem 4: (25 points)



You need to gently place a steel razor blade on a free surface of water so it "floats" on the water as shown in the figure. The razor blades come in two types with properties described below:

| Razor Type | Mass $(\mathbf{g})$ | Size (Total length of sides (mm)) |
| :--- | :--- | :--- |
| Double Edge | 0.640 | 206 |
| Single Edge | 2.61 | 154 |

(a) Derive an expression relating the surface tension to the razor blade size and mass. (15 points)
(b) Given water surface tension is $73.4 \times 10^{-3} \mathrm{~N} / \mathrm{m}$, determine which blade, if any, will float, and the corresponding contact angle. (10 points)

## Centroidal Moments of Inertia for Various Cross Sections:

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(a)

(c)

(b)

$$
\begin{aligned}
A & =\pi R^{2} \\
I_{x x} & =\frac{\pi R^{4}}{4} \\
I_{x y} & =0
\end{aligned}
$$


(d)

## Useful Equations:

Location of $x_{\mathrm{cp}}$ and $y_{\mathrm{cp}}$ of the hydrostatic force on submerged plane:

$$
\begin{aligned}
& x_{c p}=-\rho g \cdot \frac{I_{x y} \cdot \sin \theta}{F_{R}} \\
& y_{c p}=-\rho g \cdot \frac{I_{x x} \cdot \sin \theta}{F_{R}}
\end{aligned}
$$

Where $F_{\mathrm{R}}$ is the magnitude of the hydrostatic force.

Where needed, use water density of $1000 \mathrm{~kg} / \mathrm{m}^{3}$.

